Multi-Objective and Financial Portfolio Optimization of Carrier-Sense Multiple Access Protocols with Cooperative Diversity

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Abstract

This paper presents a trade-off design and optimization of a class of wireless carrier-sense multiple access protocols where collision-free transmissions are assisted by the potential cooperative retransmissions of inactive terminals with a correct copy of the original transmission. Terminals are enabled with a decode-and-forward relaying protocol. The analysis is focused on asymmetrical settings, where terminals experience different channel and queuing statistics. This work is based on multi-objective and financial portfolio optimization tools. Each packet transmission is thus regarded not only as a network resource, but also as a financial asset with different values of return and risk (or variance of the return). The objective of this financial optimization is to find the transmission policy that simultaneously maximizes return and minimizes risk in the network. The work is focused on the characterization of the boundaries (envelope) of different types of trade-off performance regions: the conventional throughput region, sum-throughput vs. fairness, sum-throughput vs. power, and return vs. risk regions. Fairness is evaluated by means of the Gini-index, which is a metric commonly used in economics to measure income inequality. Transmit power is directly linked to the global transmission rate. The protocol is shown to outperform non-cooperative solutions under different network conditions that are here discussed.

Multi-Objective and Financial Portfolio Optimization of Carrier-Sense Multiple Access Protocols with Cooperative Diversity

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Abstract. This paper addresses a trade-off design and optimization of a class of wireless carrier-sense multiple access (CSMA) protocols where collision-free transmissions are assisted by the potential cooperative retransmissions of inactive terminals with a correct copy of the original transmission. Terminals are enabled with a decode-and-forward (DF) relaying protocol. The analysis is focused on asymmetrical settings, where terminals explicitly experience different channel and queuing statistics. This work is based on multi-objective and financial portfolio optimization tools. Each packet transmission is thus considered not only as a network resource, but also as a financial asset with different values of return and risk (or variance of the return). The objective of this financial optimization is to find the transmission policy that simultaneously maximizes return and minimizes risk in the network. The work is focused on the characterization of the boundaries (envelope) of different types of trade-off performance regions: the conventional throughput region, sum-throughput vs. fairness, sum-throughput vs. power, and return vs. risk regions. Fairness is evaluated by means of the Gini-index, which is a metric commonly used in economics to measure income inequality. Transmit power is directly linked to the global transmission rate. The protocol is shown to outperform non-cooperative solutions under different network conditions that are discussed in detail in the main body of the paper.

Keywords: Cooperative diversity, random access, throughput region, multi-objective and financial portfolio optimization.

1 Introduction

1.1 Background and open issues

Wireless networks are rapidly evolving. Behind this quick evolution, there is a set of powerful, increasingly complex and adaptive physical (PHY) layer technologies that are changing the design paradigm of future networks. Advanced signal processing tools with multiple antennas, cooperative users and interference control require new cross-layer and cross-system design methodologies [1]-[3]. This means that the optimization of this advanced PHY-layer must consider medium access control (MAC) and radio resource management (RRM) issues, and vice versa, MAC and RRM algorithms should consider more details of the underlying PHY-layer. In addition, application layers are becoming more heterogeneous than ever, with different quality of service requests and different pricing policies. Different applications must be therefore processed in a different manner by the lower layers [3]. The number of metrics, parameters and issues to be simultaneously addressed is thus considerably large in comparison with legacy voice networks [4]. This already large number of metrics is expected to increase even further with the advent of cognitive radios that will allow unlicensed terminals to access underutilized portions of licensed spectrum. Each portion of the spectrum will be thus subject not only to different propagation and load conditions, but also to different licensing, billing and pricing schemes. Therefore, new tools are required in the design of future wireless networks, which are able to handle simultaneously all the involved metrics (network and economic).

1.2 Paper objectives

To partially fill this gap, this paper proposes the use of *multi-objective and financial portfolio optimization* tools for a trade-off analysis and optimization of a wireless carrier-sense multiple access (CSMA) protocol enabled with cooperative relaying diversity. Multi-objective optimization is the formal mathematical framework that addresses the simultaneous optimization of different and potentially competing objective functions or performance metrics [5]. Since this type of optimization problem usually lacks a unique solution that simultaneously satisfies the individual optimality conditions of all targeted metrics, the concept of *Pareto optimality* is commonly employed. A Pareto optimal solution provides an optimal solution for a subset of the objective functions without being dominated by any other solution [5]. The number of Pareto solutions can be potentially infinite, thus describing a Pareto trade-off front curve or surface. The objective functions of this multi-objective optimization problem can also include financial portfolio metrics such as *return* and *risk* (or variance of the return). Each network resource can be therefore considered also as a financial asset whose allocation will attempt to maximize return and minimize risk, similar to a financial stock market problem.

The system that will be subject to this multi-objective and financial portfolio optimization is a network with cooperative users. In these cooperative systems, user terminals that overheard the transmissions of other terminals in the network are allowed to relay to the base station (if necessary) copies of the original transmitted signals [6]-[9]. All the potential copies of the original signal are appropriately combined at the destination, thereby achieving gains similar to a macroscopic, virtual multiple antenna system. Cooperative relaying has gained attention over (or as complement to) other solutions such as distributed antenna systems or DAS (e.g., [10]), mainly because of the rapid and low cost potential deployment of relays in a network. Cooperative diversity has shown interesting gains in the PHY-layer that makes it suitable for future wireless networks.

However, several issues remain open in the optimization, MAC-PHY cross-layer design and RRM integration for this type of systems [9]. This paper attempts to partially fill this gap by addressing a *trade-off design, optimization and analysis* of a CSMA protocol where silent terminals are capable of relaying the signals of other terminals whose collision-free transmissions require cooperation. The original protocol and reception model were proposed previously in our works in [11] and [12], respectively. The new analysis presented here is focused on different types of trade-off region: *the conventional throughput region, sum-throughput vs. fairness, sum-throughput vs. power, and return vs. risk regions.* The results in this paper shed light on the advantages of cooperation in terms of a strict trade-off analysis between different performance metrics. Details of this analysis are included in the main body of the paper.

1.3 Related works

Techno-economic analysis and study of wireless networks has been addressed extensively in the literature. The conventional approach is the use of a techno-economic model to evaluate the revenue of an operator under a given set of resource allocation assumptions. The main objective was to find the optimum resource allocation that provides the highest revenue and that satisfies users of the network [13]. In the context of cognitive radio, research in this area has been intensive over the last few years due to the relevance of the understanding the potential gains of opportunistic spectrum usage. A review of different approaches for the use of economic optimization tools in cognitive radio can be found in [14]. The authors have also proposed a market equilibrium approach where primary and secondary users implement a learning algorithm so that they can adapt accordingly the amount of spectrum used, the pricing and the optimum demand. Most of the existing works are based on game theoretic concepts (see [15]- [19]). The work in [18] has used an atomic congestion game theoretic approach in a wireless network with spatial reuse and inter-user interference. The work in [19] addresses the problem of calculating the optimum spectrum pricing in a dynamic spectrum market. Another related approach for the use of economics in cognitive radio can be found in works such as [20] and [21] and references therein, which are based on the concepts of auction theory.

This paper uses multi-objective portfolio optimization under the assumption that each packet transmission is a financial asset. Our work explicitly introduces the concept of risk in the resource allocation problem and derives relevant expressions that allow for its interpretation as a financial stock market problem. The work in [22] has used the concept of return and variance of the return in the context of spectrum pricing. Our approach is different from these previous works regarding the explicit use of multi-objective optimization and the exploration of the boundaries of different Pareto optimal frontiers. This allows us to visualize geometrical attributes and the potential trade-off between network and economic performance metrics. In other words, instead of deriving a resource allocation policy that achieves a Nash or market equilibrium as in game theoretical works, our contribution explicitly explores the boundaries of different trade-off performance regions. In this sense, our approach complements previous works in the literature by providing a framework for trade-off analysis and explicit interpretation of financial market stock tools in wireless networks.

The structure of this paper is as follows. Section 2 describes the proposed protocol. Section 3 describes the reception model for collision-free transmissions in the presence or absence of cooperative retransmissions. Section 4 provides the definition of the performance metrics and the different trade-off performance regions. The boundaries of these trade-off regions are derived using multi-objective optimization in Section 5. Section 6 presents some performance results of the algorithm, and finally Section 7 presents the conclusions of the paper.

2 System Model and Protocol Description

Consider the slotted wireless random-access network depicted in Fig. 1 with one base station (BS) and J user terminals. Each user j has a buffer that is assumed to have always packets ready to be transmitted (full queue or dominant system assumption). All channels are independently and Rayleigh distributed with parameter σ_j for the link between user j and the BS, and with parameter $\sigma_j^{(k)}$ for the link between user j and user k. Users are allowed to cooperate with each other by relaying, if necessary, their signals towards the BS, where they are conveniently combined. The cooperative terminals will employ decode-and-forward (DF) relaying protocol. Since cooperation in half duplex systems requires more than one phase or time-slot, transmissions will be arranged in *periods or epoch-slots* with a variable length (in time-slots) denoted by the random variable l (see Fig. 1). At the beginning of an epoch-slot, each user senses the channel, and in case of being sensed as idle then the user starts a random transmission process controlled by a Bernoulli experiment with parameter p_j, which is also the transmission probability. The packet length will be fixed to L *time-slots* or *packet-units*. This means that the carrier sensing is performed L times across the duration of a transmission. Perfect carrier sensing is assumed in all derivations. All packet collisions are assumed to yield to the loss of all the transmitted information. However, whenever a collision-free transmission occurs, then all the inactive (non-contending) terminals and the BS will attempt to decode their own copy of the original collision-free signal. If the BS finds the packet as erroneous then it requests its retransmission from another terminal via an ideal feedback channel. This feedback channel has four possible outcomes $'0/1/e/r'$ which indicate, respectively, *idle* slot ($'0'$), *correct transmission* ($'1'$), *collision* ($'e'$), and *retransmission request* ($'r'$). If the feedback is $'r'$ then all the remaining idle terminals with a correct version of the original packet proceed to relay a copy in the next time-slot with probability p_R. The BS stores all the received copies and uses maximum ratio combining (MRC) with a maximum of M branches (retransmissions plus the initial transmission) to improve packet reception. Each retransmission is requested if the reception process in previous transmissions has failed. In the illustrative example in Fig. 1, the first epoch is collision-free with one cooperative retransmission. The second epoch is also collision-free but without cooperation, while the third epoch experiences an unresolvable collision.

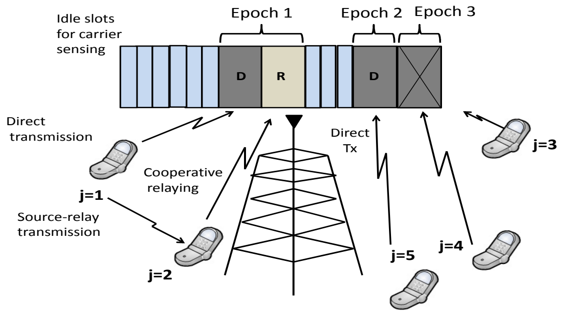

Fig. 1: Random access network assisted by carrier-sensing and cooperative diversity between terminals.

3 Packet Reception Model

This section has been mainly provided in our previous work in [11] and [12]. The results are summarized here for convenience and clarity in subsequent analysis. Consider that the instantaneous post-MRC processing SNR of user j at the BS during the nth time-slot of an epoch is denoted by $\gamma_{j,n}$. The correct reception probability of a packet of user j during the nth time-slot of an epoch, denoted by $q_{j,n}$, is given by the probability that the instantaneous SNR surpasses a packet reception threshold β [11][3]:

$$q_{j,n} = \Pr\{\gamma_{j,n} > \beta\} \tag{1}$$

Now consider that the instantaneous SNR of a transmission of user j experienced at the terminal of user k that will act as potential relay is denoted by $\gamma_j^{(k)}$ ($j \neq k$)). The correct reception probability of a packet of user j at relay k, denoted by $q_j^{(k)}$, is thus given by:

$$q_j^{(k)} = \Pr\{\gamma_j^{(k)} > \beta\}, \quad j \neq k. \tag{2}$$

Since all channels are Rayleigh distributed, then the SNR values both at the destination and at the potential relays during the first time-slot of an epoch are exponentially distributed. The reception probabilities in (1) and (2) are thus given

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[3] The SNR threshold reception model is commonly used in the literature to incorporate the effects of the PHY-layer into MAC-layer design. Therefore the instantaneous SNR is the quality indicator of the underlying channel and signal processing algorithms.

by the complementary cumulative distribution function (CCDF) of the exponential distribution $q_{j,1} = e^{-\beta/\widehat{\gamma}_{j,1}}$, and $q_j^{(k)} = e^{-\beta/\widehat{\gamma}_j^{(k)}}$, where $\widehat{\gamma}_{j,1} = E[\gamma_{j,1}] = \sigma_j^2$, $\widehat{\gamma}_j^{(k)} = E[\gamma_{j,k}] = \sigma_{j,k}^2$, and $E[\cdot]$ is the statistical average operator. Let us now address the modelling of the reception process during the cooperative phases. Since cooperative phases are activated only when the previous phases did not achieve the required SNR threshold, then it is relevant to study the statistics of reception conditional on the previous events in the preceeding time-slots. The cumulative distribution function of $\gamma_{j,n}$ conditional on the SNR of previous time slots being below β ($\gamma_{j,n-1} < \beta$) is given by see [12] for details):

$$F_{\gamma_{j,n}|\gamma_{j,n-1}<\beta}(\gamma_{j,n}) = \frac{F_{\gamma_{j,n}}(\gamma_{j,n})}{F_{\gamma_{j,n-1}}(\beta)}, \quad j \neq k., \tag{3}$$

where $F_{\gamma_{j,n}}(\gamma_{j,n})$ is the unconditional CDF of the random variable $\gamma_{j,n}$. Therefore, the reception probability during the nth time slot of an epoch given an incorrect packet reception in the previous $n-1$ transmissions is given by [12]:

$$q_{j,n|t_{j,n-1}=0} = 1 - F_{\gamma_{j,n}|\gamma_{j,n-1}<\beta}(\beta). \tag{4}$$

Details of these derivations can be found in [12].

4 Trade-off Performance Regions

4.1 Throughput region

Throughput can be defined as the long-term ratio of total number of correctly transmitted packet-units to the total number of time-slots used in the measurement. This can be proved, in our setting, to be equivalent to the ratio of the average number of correctly received *packet-units* per epoch-slot to the average length of an epoch-slot ($E[l]$). Considering that collisions yield the loss of all packets involved in the conflict, then a transmission of user j is free of collision with probability $p_j \prod_{k \neq j} \bar{p}_k$, where $\bar{a} = 1 - a$ is the complement to one, for any a (i.e.,$\bar{p}_j = 1 - p_j$). In addition, consider that $p_{s,j}$ is the correct reception probability of user j given that its transmission is collision-free and that cooperation is used. The throughput is thus given by:

$$T_j = \frac{L p_{s,j} p_j \prod_{k \neq j} \bar{p}_k}{E[l]}, \tag{5}$$

where the correct reception probability of user j in absence of collision can be otained by adding the contributions from all M possible cooperative stages:

$$p_{s,j} = q_{j,1} + \sum_{n=2}^{M} q_{j,n|t_{j,n-1}=0} \prod_{m=1}^{n-1} \bar{q}_{j,m|t_{j,m-1}=0}, \tag{6}$$

where $q_{j,m|t_{j,m-1}=0} = q_{j,1}$ when $m = 1$. The average length of an epoch-slot in the denominator of (5) can be calculated by considering all contributions of idle and busy epoch-slots: one time slot with probability $\prod_{k=1}^{J} \bar{p}_k$, at least L time-slots with probability $1-\prod_{k=1}^{J} \bar{p}_k$, and more than L time-slots with probability $\sum_j p_j \prod_{k\neq j} \bar{p}_k$ weighted by $E[l_{c,j}]$, which is the average number of cooperative retransmissions for user j once a cooperative phase has been activated. The average length of an epoch can thus be written as:

$$E[l] = \sum_{j=1}^{J} LE[l_{c,j}]p_j \prod_{k\neq j} \bar{p}_k + L + \bar{L} \prod_{k=1}^{J} \bar{p}_k, \tag{7}$$

where $E[l_{c,j}] = \sum_{n=2}^{M} (n-1) q_{j,n|t_{j,n-1}=0} \prod_{m=1}^{n-1} \bar{q}_{j,m|t_{j,m-1}=0}$. is the summation of all contributions of the M possible cooperative stages. Let us now define the concept of throughput region. For this purpose, let $\mathbf{T} = [T_1, T_2, \ldots, T_J]^T$ be the vector of stacked throughput values of all terminals, and $\mathbf{p} = [p_1, p_2, \ldots, p_J]^T$ the vector of stacked transmission probabilities. The throughput region $\mathcal{C}_T$ is the union over all possible realizations of throughput values for all terminals and for all possible transmission policies $(0 \leq p_j \leq 1)$ [23]:

$$\mathcal{C}_T = \{\tilde{\mathbf{T}} | \tilde{T}_j = T_j(p), 0 \leq p_j \leq 1\}, \tag{8}$$

which can be simply considered as the region of all achievable values of terminal throughput. The throughput region is the main performance metric used in the analysis of random access protocols in asymmetrical settings [23].

4.2 Sum-throughput vs. fairness region

The sum-throughput can be defined as follows:

$$T = \sum_{j=1}^{J} T_j. \tag{9}$$

Fairness will be evaluated in this paper by means of the Gini-index, which is commonly used in the area of economics to measure income inequality [27]. The Gini-index can be defined mathematically as follows [27]:

$$F_G = \frac{\sum_{j=1}^{M} \sum_{k=1}^{J} |T_j - T_k|}{2J^2\mu}, \tag{10}$$

where $\mu = \sum_{j=1}^{J} T_j / J$ is the mean value. A value of Gini-index of zero indicates the best fairness scenario where the users have identical statistical performance. A value of F_G equal to one is the worst fairness scenario as only one user overtakes all the resources of the network. For convenience in subsequent analysis, (10) can be rewritten as follows: $F_G = \frac{\sum_{j=1}^{J} \sum_{k=1}^{J} a_{j,k}(T_j - T_k)}{2JT} = \frac{X}{2JT}$ where $a_{j,k}$ is defined as $a_{j,k} = \begin{cases} 1, & T_j \geq T_k \\ -1, & T_j < T_k \end{cases}$. Consider the vector $\mathbf{F} = [T \quad F_G]^T$ of stacked values of sum-throughput and fairness. The sum-throughput vs. fairness trade-off region can be defined as the union of all achievable values $[T \quad F_G]$ for all possible transmission policies ($0 \leq p_j \leq 1$):

$$\mathcal{C}_F = \{\tilde{\mathbf{F}} | \tilde{T} = T(\mathbf{p}), \tilde{F}_G = F_G(\mathbf{p}), 0 \leq p_j \leq 1\}. \tag{11}$$

4.3 Sum-throughput vs. transmit power region

In this paper, average transmit power will be considered as proportional to the transmit rate of the system plus the potential cooperative retransmissions. Therefore, in our setting, we can define the average consumed power as follows:

$$P = \alpha \sum_{j=1}^{J} p_j (1 + E[l_{c,j}]), \tag{12}$$

where α is a proportionality constant. Having defined both sum-throughput and transmit power consumption, let us now define the concept of sum throughput vs power trade-off region. First, we define the vector $\mathbf{P} = [T \quad P]^T$ of stacked values of sum-throughput and power. The sum-throughput vs. power trade-off region can be defined as the union of all achievable values $[TP]$ for all possible transmission policies ($0 \leq p_j \leq 1$):

$$\mathcal{C}_P = \{\tilde{\mathbf{P}} | \tilde{T} = T(\mathbf{p}), \tilde{P} = P(\mathbf{p}), 0 \leq p_j \leq 1\}. \tag{13}$$

4.4 Return vs. risk trade-off region

Let us define the instantaneous return per correctly transmitted packet of user j as r_j, and the average return as $E[r_j] = \hat{r}_j$. The instantaneous return of the network per epoch-slot can be thus written as follows:

$$R = \sum_{j=1}^{J} r_j t_j, \tag{14}$$

where t_j is the binary random variable that indicates whether a packet transmission was correct or not per epoch-slot. The average return can be thus defined as the ratio of the average return per epoch-slot to the the average length of an epoch-slot:

$$\hat{R} = \frac{E[R]}{E[l]} = \sum_{j=1}^{J} \hat{r}_j T_j. \tag{15}$$

Let us now calculate the average risk as the ratio of the variance of the instantaneous return per epoch to the average length of an epoch:

$$S = \frac{E[R^2] - E[R]^2}{E[l]} = \frac{E[R^2]}{E[l]} - E[l]\hat{R}^2 = \frac{\sum_{j=1}^{J} E[r_j^2]}{E[l]} - E[l]\hat{R}^2 \tag{16}$$

Consider the vector $\mathbf{R} = [\hat{R} \quad S]^T$ of stacked values of return and risk. The return vs risk trade-off region can be defined as the union of all achievable values $[\hat{R} \quad S]$ for all possible transmission policies ($0 \leq p_j \leq 1$) :

$$\mathcal{C}_R = \{\tilde{\mathbf{R}} | \tilde{R} = R(\mathbf{p}), \tilde{S} = S(\mathbf{p}), 0 \leq p_j \leq 1\}. \tag{17}$$

5 Multi-objective Optimization

To obtain the envelope of the trade-off regions, a multi-objective optimization of I functions F_i is here proposed:

$$\mathbf{p}_{opt} = \arg\max_{\mathbf{p}} \quad [F_1, F_2 \quad \ldots F_i, \ldots \quad F_I]. \tag{18}$$

Since this vector optimization usually lacks a unique solution [5], the concept of Pareto optimal trade-off front is commonly employed. A Pareto optimal solution can be loosely defined here as the point that is at least optimum for one or more of the elements of the vector objective function $[F_1, F_2 \quad \ldots \quad F_I]$, or in other words when none of the objective functions can be improved in value without degrading some of the other objective values (see [5] for a complete definition). The multi-objective optimization problem can be transformed into a single objective optimization problem using the method of scalarization [5]:

$$\mathbf{p}_{opt} = \arg\max_{\mathbf{p}} \sum_i \mu_i F_i, \tag{19}$$

where μ_i is the relative weight given to the ith objective function. Differentiating the objective function in (19) we obtain a set of equations given by $\sum_i \mu_i \frac{\partial F_i}{\partial p_k} = 0, \quad k = 1.., J$. Assuming $J \geq I$, the solution of a subset $\mathcal{S}_o$ of I of these linear equations independent from the values of the weighting factors μ_k can be proved, in our context, to be equivalent to setting the following Jacobian determinant to zero [25] [24]:

$$|\mathbf{J}_o| = 0, \tag{20}$$

where $J_o(k,i) = \frac{\partial F_i}{\partial p_k}$ is the (i,k) entry of the Jacobian matrix $\mathbf{J}_o$, $k \in \mathcal{S}_o$. The final solution is given by the union of the solutions for all the possible selections of equations $\mathcal{S}_o$.

5.1 Throughput Region

In the case of the throughput region, the $I = J$ objective functions to be optimized in (19) are the throughput functions of each terminal: $F_j = T_j$, $j = 1, \ldots, J$. This means that the elements of the Jacobian determinant in (20) are given by $J_{k,j} = \frac{\partial T_k}{\partial p_j}$. In this case, the number of objective functions is equivalent to the number of variables of the optimization. The final expression is given by (see [11] for details of the derivation):

$$\sum_{j=1}^{J} L p_j = L + \bar{L} \prod_{j=1}^{J} \bar{p}_j. \tag{21}$$

This last expression together with the expression for the throughput of the different users in (5) characterize the boundary of the throughput region.

5.2 Sum-throughput vs Fairness

In the case of the sum-throughput vs. fairness, the $I = 2$ objective functions to be optimized are $F_1 = T$ in (5) and $F_2 = F_G$ in (10). Therefore, the Jacobian determinant in (20) reduces to:

$$\frac{\partial X}{\partial p_k}\frac{\partial T}{\partial p_j} = \frac{\partial X}{\partial p_j}\frac{\partial T}{\partial p_k}, \quad j \neq k, \quad j, k \in \{1, \ldots, J\}. \tag{22}$$

In the particular case of two users $J = 2$ the previous expression can be proved to be equivalent to the Jacobian of the throughput region and thus boil down to the solution in (21). Further details are provided in the section of results.

5.3 Sum-throughput vs. transmit power region

In the case of the sum-throughput vs. power, the $I = 2$ objective functions to be optimized are $F_1 = T$ in (5) and $F_2 = P$ in (12). Therefore, the Jacobian determinant in (20) reduces to

$$\begin{vmatrix} \frac{\partial P}{\partial p_j} & \frac{\partial P}{\partial p_k} \\ \\ \frac{\partial T}{\partial p_j} & \frac{\partial T}{\partial p_k} \end{vmatrix} = 0, \quad j \neq k, \quad j, k \in \{1, \ldots, J\}. \tag{23}$$

This expression can be solved via numerical methods as explained in the next section.

5.4 Return vs. risk

In the case of the return vs. risk trade-off region, the $I = 2$ objective functions to be optimized are $F_1 = R$ in (15) and $F_2 = S$ in (16). Therefore, the Jacobian determinant in (20) becomes:

$$\begin{vmatrix} \frac{\partial \hat{R}}{\partial p_j} & \frac{\partial S}{\partial p_k} \\ \\ \frac{\partial \hat{R}}{\partial p_j} & \frac{\partial S}{\partial p_k} \end{vmatrix} = 0, \quad j \neq k, \quad j, k \in \{1, \ldots, J\}. \tag{24}$$

6 Results

This section presents the graphical results of the analytic work presented in previous sections. For convenience, all results assume a system with $J = 2$ users. All the results can be easily extended to a multi-user scenario. User 1 will be modelled with low reception probabilities using a parameter $\hat{\gamma}_{1,1} = 1$, while the second user will experience high values of reception probabilities with parameter $\hat{\gamma}_{2,1} = 10$. User-to-user communication is implemented with parameter $\hat{\gamma}_1^{(2)} = \hat{\gamma}_2^{(1)} = 8$. The reception threshold is set to $\beta = 1$. In terms of financial parameters,

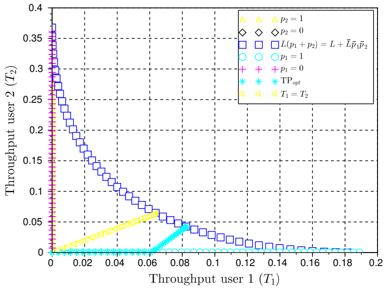


Fig.2: Throughput region for $L = 1$ and $M = 1$.

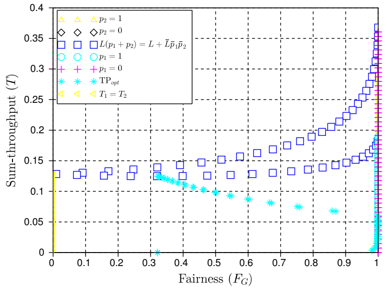


Fig.3: Sum-throughput (T) vs. fairness (F_G) region for $L = 1$ and $M = 1$

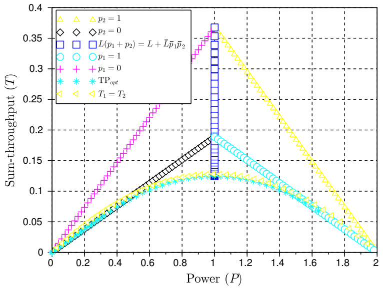


Fig.4: Sum-throughput (T) vs. Tx. power (P) region for $L = 1$ and $M = 1$.

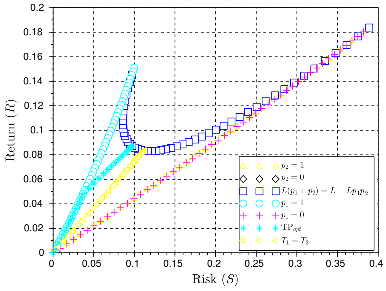


Fig.5: Return ($\hat{R}$) vs. risk (S) region for $L = 1$ and $M = 1$.

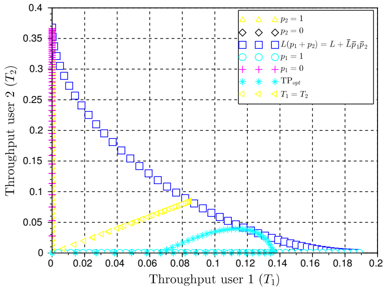


Fig.6: Throughput region for $L = 4$ and $M = 1$.

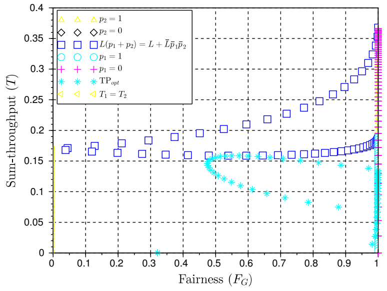
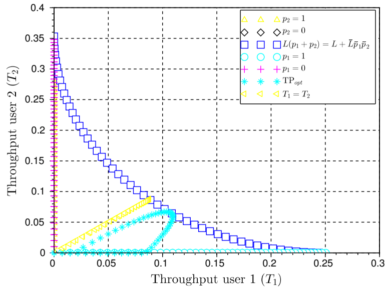


Fig.7: Sum-throughput (T) vs. fairness (F_G) region for $L = 4$ and $M = 1$.



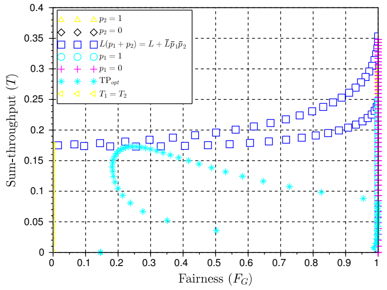
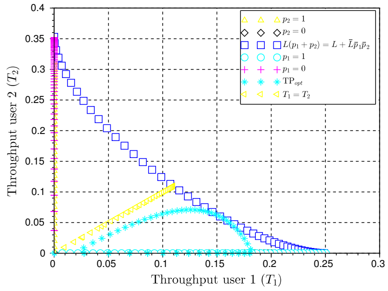


Fig.8: Throughput region for $L = 1$ and $M = 4$.

Fig.9: Sum-throughput (T) vs. fairness (F_G) for $L = 1$ and $M = 4$.



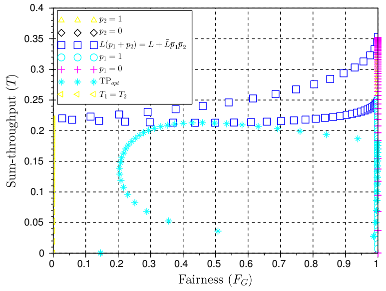
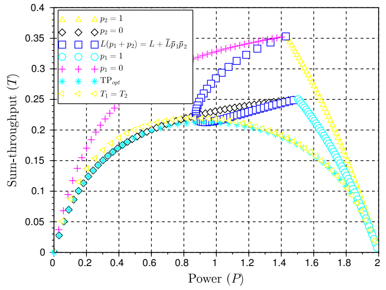
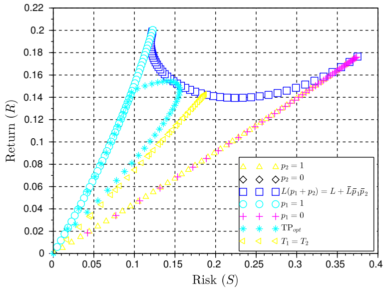


Fig.10: Throughput region for $L = 4$ and $M = 4$.

Fig.11: Sum-throughput (T) vs. fairness (F_G) region for $L = 4$ and $M = 4$.





Sum-throughput (T) vs. Tx. power (P).

Return ($\hat{R}$) vs. risk (S).

we selected $\hat{r}_1 = 0.8$, $\hat{r}_2 = 0.5$, $E[r_1^2] = 0.01$, and $E[r_2^2] = 0.9$. While this is a rather arbitrary selection of financial parameters, it is possible to obtain some useful results and conclusions for the general case.

Fig. 2 to Fig. 5 present the sketches of different trade-off regions for the case of $L = 1$ and $M = 1$, which is a random access protocol without carrier-sensing (ALOHA) and without cooperative diversity. All the figures contain the envelopes of the different trade-off regions obtained from multi-objective optimization, relevant boundary conditions (e.g. $p = 0$, $p = 1$), and also the projections of the boundaries(envelopes) of the other regions under analysis. Fig. 2 shows the throughput region, where we can observe it is non-convex due to the collision model and the ALOHA protocol operation. The boundary described by the Pareto solution is labelled as $L(p_1 + p_2) = L + \bar{L}\bar{p}_1\bar{p}_2$ which is the expression in (21) for a two-user system. The projections of the Pareto optimal throughput-power envelope (labelled TP_{opt}) and the equal throughput curve (labelled $T_1 = T_2$) are also displayed. The non-convexity of the throughput region means that the trade-off region of sum-throughput and fairness region in Fig. 3 exhibits a rapid decrease of throughput for increasingly improving values of Gini-index (a value of zero indicates the best fairness condition). Note also that the boundary of the fairness region is described by half of the solution that describes the throughput region, which is the half corresponding to the user with best channel conditions. The other half is also displayed inside the region in Fig. 3. The sum-throughput vs. power trade-off region displayed in Fig. 4 shows that the region is defined by boundary conditions and by the Pareto solution in (23), which also describes the minimum sum-throughput curve. Note that the Pareto front of the throughput region labelled as $L(p_1 + p_2) = L + \bar{L}\bar{p}_1\bar{p}_2$ is projected as a vertical constant power line that cuts the region into two equal halves. The return vs. risk trade-off region displayed in Fig.5 is defined by boundary conditions with the point of maximum and minimum risk. The curve that defines the Pareto solution for the throughput and fairness region also describes the Pareto solution of the return vs. risk region by joining the points of maximum or minimum return (or risk). The non-convexity of the throughput region makes the return vs. risk region also non-convex, where it is difficult to achieve high values of return without compromising risk and also fairness.

Fig. 6 and Fig.7 show, respectively, the throughput and sum-throughput vs. fairness trade-off regions for the case of $L = 4$ and $M = 1$, which is a carrier-sensing algorithm without cooperation. We can observe that the throughput region has become less non-convex, which leads to an increase of its area. This improvement on the convexity of the region can be also observed in a reduction of the steepness of the fairness Pareto curve in Fig. 7, which means that an improvement on fairness (reduction of Gini-index) does not yield a large drop of sum-throughput as in the case of ALOHA discussed previously in Fig 2 to Fig. 5.

Fig. 8 and Fig. 9 show, respectively, the throughput and sum-throughput vs fairness trade-off regions for the case of $L = 1$ and $M = 4$, which is an ALOHA protocol enabled with cooperative diversity. We can observe that the throughput region has become less non-convex, but not as much as in the previous case with carrier-sensing, also yielding an increase of its area. We can observe that the increase of the area due to cooperation is mainly due to the improvement of the reception probabilities, which makes the user with the lower reception probability take benefit from the improved relaying capabilities. Note that, unlike the case of pure carrier-sensing displayed in Fig. 6 the region displayed in Fig. 8 with cooperation provides an effective improvement of the reception probability, particularly for user 1, shifting the area slightly towards the right hand side of the figure. By contrast, carrier sensing seems to improve the region mainly at the middle of the trade-off boundary region, which is the zone dominated by collisions. From these observations we can therefore conclude that both carrier sensing and cooperation yield useful improvements on the operation of the protocol under different networking circumstances: carrier sensing improves the avoidance of collisions, while cooperation improves the effective reception capabilities of the system. This improvement on the convexity of the region can be also observed in a reduction of the steepness of the fairness Pareto curve in Fig. 9, which means that an improvement on fairness (reduction of Gini index) is not accompanied by a considerable decline of aggregate throughput.

Fig. 10 to Fig. 13 show the results for a system with $L = 4$ and $M = 4$ combining the benefits from carrier-sensing and cooperation. Observe that the throughput region is considerably increased with a less non-convex shape, which is the result of improved collision management with carrier sensing and also improved reception probability due to cooperative relaying. These improvements are also translated into a better trade-off between fairness and sum-throughput which are shown as a more flat curve in Fig. 11. Higher values of sum-throughput can be achieved without sacrificing too much fairness between users. In terms of power consumption we can observe in Fig. 12 that power consumption has been considerably increased in comparison to the ALOHA case without cooperation. However, we can observe that the increase of power consumption along the curve labelled with $L(p_1 + p_2) = L + \bar{L}\bar{p}_1\bar{p}_2$ is mainly for the case where one of the users transmits while the other user is idle, and that even power reduction can be observed in the region where both users start contending with each other. Therefore, we can conclude that cooperation and carrier-sensing can achieve good levels of sum-throughput without compromising too much consumed power and fairness. In terms of financial performance we can observe in Fig. 13 that higher levels of return can be achieved with a good level of risk, in comparison with previous result in Fig. 5. This means that risk has been effectively reduced by means of cooperation and carrier-sensing.

7 Conclusions

This paper has presented the MAC-PHY cross-layer design of a class of carrier-sense multiple access protocol where users with good channel states can cooperate with users with bad channel states by relaying a copy of collision-free signals. Different types of trade-off region were here analysed by means of multi-objective and financial convex optimization tools. It was confirmed that cooperation provides an improvement of the reception capabilities of the system, particularly for users with bad channels states which benefit from users with better channel states relaying their signals towards the base station, where they were conveniently combined. This improved reception was translated in an increase of the throughput region, reduced steepness of the Pareto curve of sum-throughput vs fairness and a better trade-off between return and risk in the network. In terms of power consumption, cooperation provides a considerable increase but in combination with carrier sensing was proved to yield a good compromise between network performance and consume power. Carrier-sensing was proved to reduce the non-convexity of the throughput region particularly when both users contend for the channel, which is also translated in a better trade-off between sum-throughput and fairness. In combination with cooperative diversity, carrier-sensing proved to yield to a considerable increase also in terms of the stability region of the algorithm. Future work includes the use of multi-objective and financial optimization tools in the analysis of more complex random access schemes.

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